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3 (Sem–3/CBCS) MAT HC 3

2021

(Held in 2022)

MATHEMATICS

(Honours)

Paper : MAT–HC–3036

(Analytical Geometry)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×10=10

   (i) What is the nature of the conic represented by

   $$4x^2 - 4xy + y^2 - 12x + 6y + 9 = 0 \ ?$$

   (ii) Define skew lines.

Contd.

(iii) Under what condition

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

may represents a pair of parallel straight lines ?

(iv) If the axes are rectangular, find the direction cosines of the normal to the plane $x + 2y - 2z = 9$.

(v) Write down the conditions under which the general equation of second degree

$$ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$$

represents a sphere.

(vi) If $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ is a generator of the cone represented by the homogeneous equation $f(x, y, z)$, then what is the value of $f(l, m, n)$?

(vii) What is meant by diametral plane of a conicoid ?

(viii) Find the equation of the line $\frac{x}{a}+\frac{y}{b}=2$, when the origin is transferred to the point (a, b).

(ix) Find the point on the conic $\frac{8}{r}=3-\sqrt{2}\cos\theta$ whose radius vector is 4.

(x) What is the polar equation of a circle when the pole is at the centre ?

2. Answer the following questions : 2×5=10

(a) Write down the equation to the cone whose vertex is the origin and which passes through the curve of intersection of the plane $lx+my+nz=p$ and the surface $ax^2+by^2+cz^2=1$.

(b) Transform the equation $x^2-y^2=a^2$ by taking the perpendicular lines $y-x=0$ and $y+x=0$ as coordinate axes.

(c) If $\left(at_1^2, 2at_1\right)$ and $\left(at_2^2, 2at_2\right)$ are the extremities of any focal chord of the parabola $y^2 = 4ax$, then prove that $t_1 t_2 = -1$.

(d) Find the centre and foci of the hyperbola $x^2 - y^2 = a^2$.

(e) Find where the line $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4}$ meets the plane $x + y + z = 3$.

3. Answer ***any four*** : 5×4=20

(a) If by transformation from one set of rectangular axes to another with the same origin the expression $ax + by$ changes to $a'x' + b'y'$, prove that $a^2 + b^2 = a'^2 + b'^2$.

(b) Prove that the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a pair of parallel straight lines, if $\frac{a}{h} = \frac{h}{b} = \frac{g}{f}$.

(c) Find the condition that line

$$\frac{l}{r} = A\cos\theta + B\sin\theta$$

may touch the conic $\frac{l}{r} = 1 - e\cos\theta$.

(d) Find the equation to the plane which cuts $x^2 + 4y^2 - 5z^2 = 1$ in a conic whose centre is the point (2,3,4).

(e) Show that the equation to the cone whose vertex is origin and base is $z = k, f(x, y) = 0$ is $f\left(\frac{kx}{z}, \frac{ky}{z}\right) = 0$.

(f) A variable plane is at a constant distance p from the origin and meets the axes, which are rectangular in A, B, C. Through A, B, C planes are drawn parallel to the coordinate planes, show that locus of their point of intersection is given by $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.

4. Answer the following questions : 10×4=40

(a) Find the point of intersection of the lines represented by the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.

(b) Show that the equation
$$9x^2 - 24xy + 16y^2 - 18x - 101y + 19 = 0$$
represents a parabola and it can be reduced to the standard form $Y^2 = 3X$. Find the coordinates of the vertex and the focus.

(c) Prove that the sum of the reciprocals of two perpendicular focal chords of a conic is constant.

(d) Show that the ortho-centre of the triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my = 1$ is given by $\frac{x}{l} = \frac{y}{m} = \frac{a+b}{am^2 - 2hlm + bl^2}$

(e) Find the condition that the plane $lx + my + nz = p$ may touch the conicoid $ax^2 + by^2 + cz^2 = 1$. Verify that the plane $2x - 2y + 8z = 9$ touches the ellipsoid $x^2 + 2y^2 + 3z^2 = 9$.

(f) Show that the shortest distance between any two opposite edges of the tetrahedron formed by the planes $y + z = 0$, $z + x = 0$, $x + y = 0$, $x + y + z = a$ is $\frac{2a}{\sqrt{6}}$ and that the three lines of shortest distance intersect at the point $x = y = z = -a$.

(g) Find the equation to the cylinder generated by the lines drawn through the points of the circle

$x+y+z=1, x^2+y^2+z^2=4$ which are

parallel to the line $\frac{x}{2}=\frac{y}{-1}=\frac{z}{2}$.

(h) A variable plane is parallel to the given

plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$ and meets the axes

in A, B, C respectively. Prove that the circle ABC lies on the cone

$$yz\left(\frac{b}{c}+\frac{c}{b}\right)+zx\left(\frac{c}{a}+\frac{a}{c}\right)+xy\left(\frac{a}{b}+\frac{b}{a}\right)=0.$$
