3 (Sem–5/CBCS) MAT HE 4/5/6

2021

(Held in 2022)

MATHEMATICS

(Honours Elective)

Answer the Questions from any one Option.

OPTION–A

Paper : MAT–HE–5046

(Linear Programming)

DSE(H)–2

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : $1\times10=10$

   (a) A basic feasible solution whose variables are.

   (i) degenerate

   (ii) nondegenerate

(iii) non-negative

(iv) None of the above

(Choose the correct answer)

(b) The inequality constraints of an LPP can be converted into equation by introducing

(i) negative variables

(ii) non-degenerate B.F.

(iii) slack and surplus variables

(iv) None of the above

(Choose the correct answer)

(c) A solution of an LPP, which optimize the objective function is called

(i) basic solution

(ii) basic feasible solution

(iii) optimal solution

(iv) None of the above

(Choose the correct answer)

(d) What is artificial variable of an LPP ?

(e) Write the equation of line segment in $\mathbb{R}^n$.

(f) Define dual of a given LPP.

(g) What is pure strategy of game theory ?

(h) Is region of feasible solution to an LPP constitute a convex set ?

(i) Is every convex set in $\mathbb{R}^n$ a convex polyhedron also ?

(j) Is every boundary point an extreme point of a convex set ?

2. Answer the following questions : 2×5=10

(a) Show that the feasible solution $x_1 = 1, x_2 = 0, x_3 = 1, z = 6$ to the system

$$\min \ Z = 2x_1 + 3x_2 + 4x_3$$

s.t. $x_1 + x_2 + x_3 = 2$

$x_1 - x_2 + x_3 = 2 \ , \ x_i \geq 0$

is not basic.

(b) A hyperplane is given by the equation

$$3x_1 + 2x_2 + 4x_3 + 7x_4 = 8$$

Find in which half space do the point $(-6, 1, 7, 2)$ lie.

(c) Find extreme points if any of the set $S = \{(x, y) : |x| \leq 1, |y| \leq 1\}$

(d) Show by an example that the union of two convex sets is not necessarily a convex set.

(e) If $x_1 = 2,\ x_2 = 3,\ x_3 = 1$ a BFS of the LPP

$$\max\ Z = x_1 + 2x_2 + 4x_3$$

$$\text{s.t.}\quad 2x_1 + x_2 + 4x_3 = 11$$

$$3x_1 + x_2 + 5x_3 = 14$$

$$x_1, x_2, x_3 \geq 0$$? Explain.

3. Answer ***any four*** questions : 5×4=20

(a) Prove that the set of all feasible solutions of an LPP is a convex set.

(b) Sketch the convex polygon spanned by the following points in a two-dimensional Euclidean space. Which of these points are vertices ? Express the other as the convex linear combination of the vertices

$$(0,0),\ (0,1),\ (1,0),\ \left(\frac{1}{2},\frac{1}{4}\right).$$

(c) If $x_0 \in S$ where S is the set of all FS of the LPP min $Z = cx$, such that $Ax = b$, $x \geq 0$ minimize the objective function $Z = cx$, then show that x_0 also maximize the objective function $Z^* = (-c)x$ over S.

(d) Find the dual of the following LPP :

$\min Z_p = x_1 + x_2 + x_3$

s.t.

$$x_1 - 3x_2 + 4x_3 = 5$$
$$2x_1 - 3x_2 \leq 3$$
$$2x_2 - x_3 \geq 5$$
$$x_1, x_2, x_3 \geq 0$$

(e) Prove that the dual of a dual is a primal problem itself.

(f) Write the characteristics of an LPP in canonical form.

4. Answer *(a)* **or** *(b)*, *(c)* **or** *(d)*, *(e)* **or** *(f)*, *(g)* **or** *(h)* : 10×4=40

(a) Old hens can be bought for Rs. 2 each but young ones cost Rs. 5 each. The old hens lay 3 eggs per week and the young ones 5 eggs per week, each being worth 30 paise. A hen costs Re. 1 per week to feed. If I have only Rs. 80 to spend for hens, how many of each kind shall I buy to give a profit of more than Rs. 6 per week, assuming that I can not house more than 20 hens ? Formulate the LPP and solve by graphical method.

(b) Find all basic and then all the basic feasible solutions for the equations

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$
$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$$

and determine the associated general convex combination of the extreme point solutions.

(c) State and prove the fundamental theorem of LPP.

(d) Solve by simplex method :

$$\max \quad Z = 3x_1 + 5x_2 + 4x_3$$
$$\text{s.t.} \quad 2x_1 + 3x_2 \le 8$$
$$3x_1 + 2x_2 + 4x_3 \le 15$$
$$2x_2 + 5x_3 \le 10$$
$$x_1, x_2, x_3 \ge 0$$

(e) If in an assignment problem, a constant is added or substracted to every element of a row (or column) of the cost matrix $[c_{ij}]$, then prove that an assignment which minimizes the total cost for one matrix, also minimizes the total cost for the other matrix.

(f) Solve the following transportation problem :

To

From		S_1	S_2	S_3	S_4	Supply
	O_1	1	2	1	4	30
	O_2	3	3	2	1	50
	O_3	4	2	5	9	20
Demand		20	40	30	10	100

(g) For any zero-sum two-persons game where the optimal strategies are not pure and for which A's pay-off matrix is

		B	
		I y_1	II y_2
A	x_1 I	a_{11}	a_{12}
	x_2 II	a_{21}	a_{22}

the optimal strategies are (x_1, x_2) and (y_1, y_2) then prove that

$$\frac{x_1}{x_2} = \frac{a_{22} - a_{21}}{a_{11} - a_{12}} \text{ and } \frac{y_1}{y_2} = \frac{a_{22} - a_{12}}{a_{11} - a_{21}} \text{ and}$$

the value of the game to A is given by

$$v = \frac{a_{11}a_{22} - a_{12}a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

(h) Solve the game whose pay-off matrix is

$$\begin{bmatrix} -1 & -2 & 8 \\ 7 & 5 & -1 \\ 6 & 0 & 12 \end{bmatrix}$$
