3 (Sem–6/CBCS) MAT HC 2

2022

MATHEMATICS

(Honours)

Paper : MAT–HC–6026

(Partial Differential Equations)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any seven*** : 1×7=7

   (i) The equation of the form $P_p + Q_q = \mathbb{R}$ is known as

   (a) Charpit's equation

   (b) Lagrange's equation

   (c) Bernoulli's equation

   (d) Clairaut's equation

   (Choose the correct answer)

(ii) How many minimum no. of independent variables does a partial differential equation require?

(iii) Find the degree and order of the equation

$$\frac{\partial^3 z}{\partial x^3}+\left(\frac{\partial^3 z}{\partial x \partial y^2}\right)^2+\frac{\partial z}{\partial y}=\sin(x+2y)$$

(iv) Which method can be used for finding the complete solution of a non-linear partial differential equation of first order

(a) Jacobi method

(b) Charpit's method

(c) Both (a) and (b)

(d) None of the above

(Choose the correct answer)

(v) State *True* **Or** *False* :

The equation

$$u_{xx}+u_{yy}+u_{zz}=0$$

is an Hyperbolic equation.

(vi) Fill in the blanks :

$$\left(\frac{\partial z}{\partial x}\right)^2+2\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}+z=0$$

is a ________ ________ order partial differential equation.

(vii) The characteristic equation of $yu_x + xu_y = u$ is

(a) $\frac{dx}{x} = \frac{dy}{y} = \frac{du}{u}$

(b) $\frac{dx}{y} = \frac{dy}{x} = \frac{du}{u}$

(c) $\frac{dx}{u} = \frac{dy}{x} = \frac{du}{y}$

(d) None of the above

(Choose the correct answer)

(viii) State *True* **Or** *False*

$xu_x + yu_y = u^2 + x^2$ is a semi-linear partial differential equation.

(ix) Fill in the blanks :

A solution $z = z(x, y)$ when interpreted as a surface in 3-dimensional space is called ________ ________.

(x) The partial differential equation is elliptical if

(a) $B^2 - 4AC > 0$

(b) $B^2 - 4AC \geq 0$

(c) $B^2 - 4AC \leq 0$

(d) $B^2 - 4AC < 0$

(Choose the correct answer)

2. Answer ***any four*** : 2×4=8

(i) Define quasi-linear partial differential equation and give *one* example.

(ii) Show that a family of spheres $(x-a)^2+(y-b)^2=r^2$ satisfies the partial differential equation

$$z^2\left(p^2+q^2+1\right)=r^2$$

(iii) Eliminate the constants a and b from $z=(x+a)(y+b)$.

(iv) Determine whether the given equation is hyperbolic, parabolic or elliptic

$$u_{xx}-2u_{yy}=0.$$

(v) Solve the differential equation $p+q=1$.

(vi) Explain the essential features of the "Method of separation of variables".

(vii) Mention when Charpit's method is used. Name a disadvantage of Charpit's method.

(viii) What is the classification of the equation

$$u_{xx}-4u_{xy}+4u_{yy}=e^y$$

3. Solve ***any three*** : 5×3=15

(i) Form a partial differential equation by eliminating arbitrary functions f and F from $y = f(x - at) + F(x + at)$.

(ii) Solve

$$y^2 p - xyq = x(z - 2y)$$

(iii) Find the integral surface of the linear partial differential equation

$$x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$$

which contains the straight line $x + y = 0$, $z = 1$.

(iv) Find the solution of the equation $z = pq$ which passes through the parabola $x = 0$, $y^2 = z$.

(v) Find a complete integral of the equation $x^2p^2 + y^2q^2 = 1$.

(vi) Reduce the equation $yu_x + u_y = x$ to canonical form and obtain the general solution.

(vii) Apply the method of separation of variables $u(x, y) = f(x)\, g(y)$ to solve the equation $u_x + u = u_y$, $u(x, 0) = 4e^{-3x}$.

(viii) Determine the general solution of $4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2$.

4. Answer ***any three*** : 10×3=30

(i) Solve $(p^2 + q^2)y - qz = 0$ by Jacobi method.

(ii) Solve $z^2 = pqxy$ by Charpit's method.

(iii) Find the general solution of the differential equation

$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x + y)z$$

(iv) Solve

$$(mz - ny)p + (nx - lz)q = ly - mx$$

(v) Use $v = \ln u$ and $v = f(x) + g(y)$ to solve the equation

$$x^2 u_x^2 + y^2 u_y^2 = u^2.$$

(vi) Find the solution of the equation

$$z = \frac{1}{2}\left(p^2 + q^2\right) + (p - x)(q - y)$$

which passes through the x axis.

(vii) Find the canonical form of the equation

$$y^2 u_{xx} - x^2 u_{yy} = 0.$$

(viii) Classify the second order linear partial differential equation with example.
